

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

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Time 1 hour 30 minutes

Paper
reference

WMA12/01

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:

Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ►

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Pearson

1. Adina is saving money to buy a new computer. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the computer.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

- (a) (i) state with a reason whether an arithmetic series or a geometric series should be used,
- (ii) write down an expression, in terms of n , for the amount, in pounds (£), saved in week n .

(3)

Given that the computer Adina wants to buy costs £350

- (b) find the number of weeks it will take for Adina to save enough money to buy the computer.

(4)

a.i. There is a common difference, not a common ratio, so an arithmetic sequence should be used. ✓ B1

ii. NTH TERM OF ARITHMETIC SEQUENCE

$$a_n = a + (n-1)d$$

$$a = 5 \text{ (first term)}$$

$$d = 5.25 - 5 = 0.25$$

$$a_n = 5 + 0.25(n-1) \quad \checkmark M1$$

$$a_n = 5 + 0.25(n-1) \quad \checkmark A1$$

- b. We need the sum of the arithmetic series to be equal to 350

ARITHMETIC SERIES

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$\frac{n}{2}(2 \times 5 + 0.25(n-1)) = 350 \quad \checkmark M1 \quad \checkmark A1$$

$$10n + 0.25n(n-1) = 700$$

$$10n + 0.25n^2 - 0.25n = 700$$

$$n^2 + 39n - 2800 = 0$$

QUADRATIC FORMULA

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-39 \pm \sqrt{(-39)^2 - 4(1)(-2800)}}{2 \times 1} \quad \checkmark M1$$



Question 1 continued

$n = 36.89\dots$

$n = 37$

37 weeks ✓ A1

Q1

(Total 7 marks)



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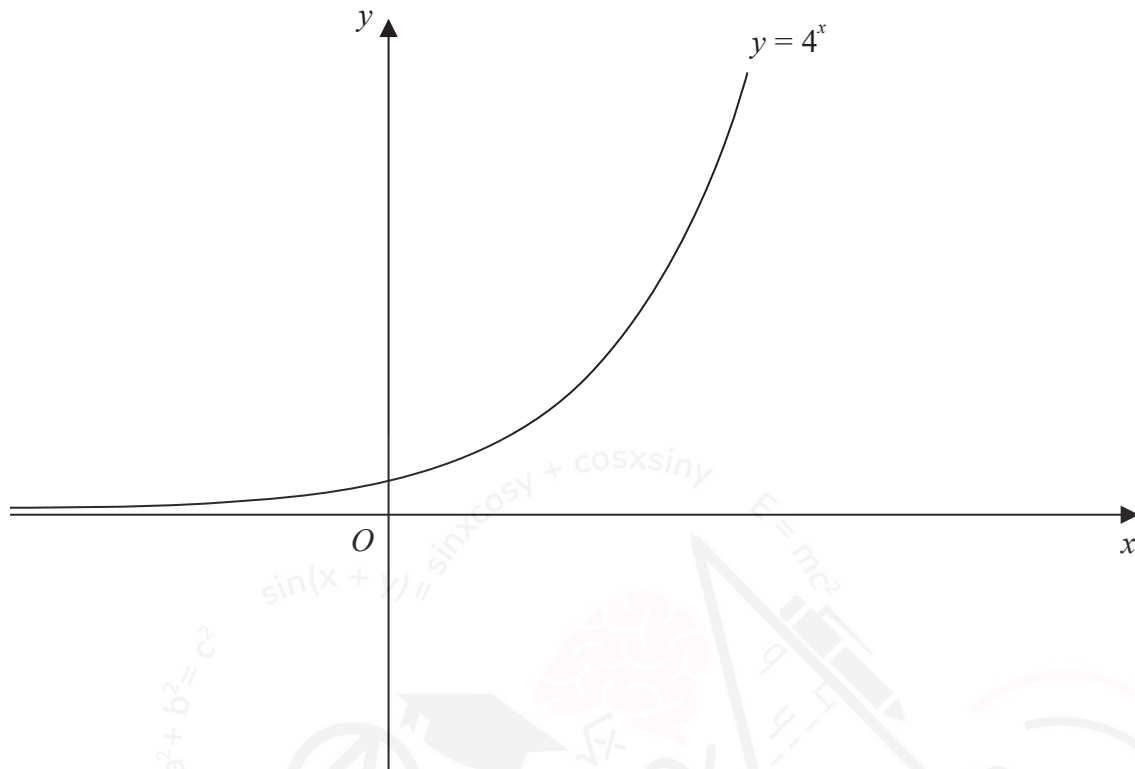
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = 4^x$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(a) On Diagram 1, sketch the curve with equation

(i) $y = 2^x$

(ii) $y = 4^x - 6$

Label clearly the coordinates of any points of intersection with the coordinate axes.

(4)

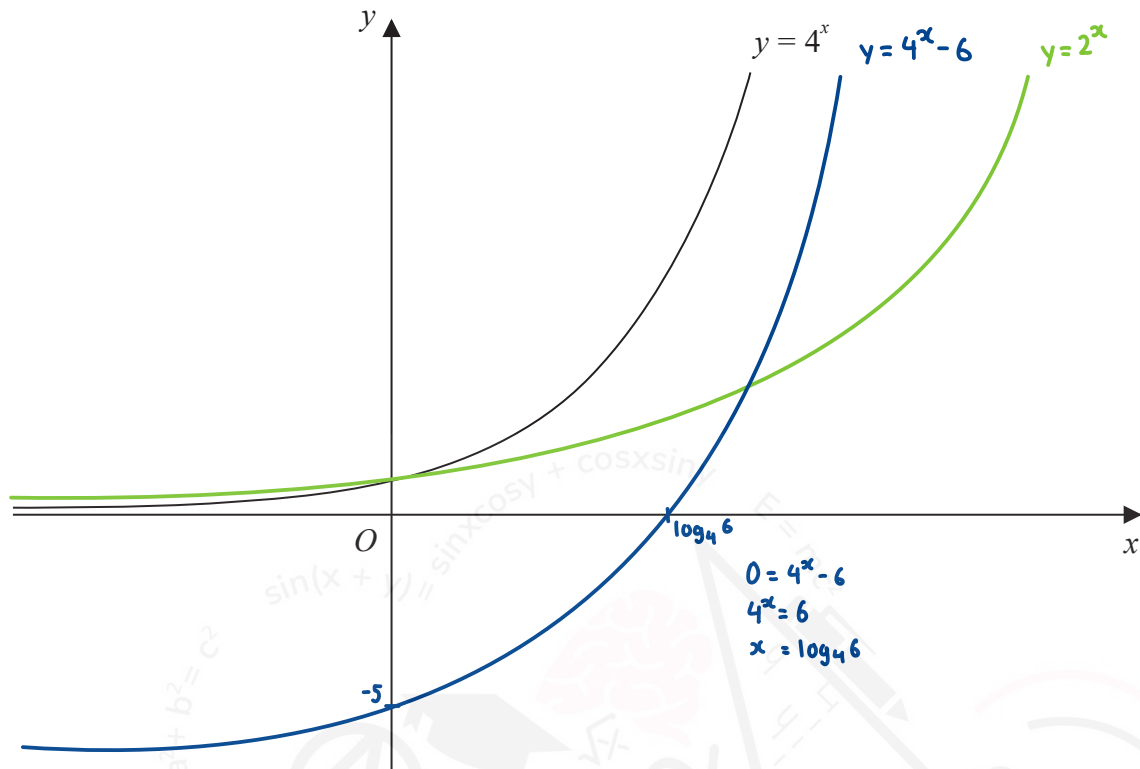
The curve with equation $y = 2^x$ meets the curve with equation $y = 4^x - 6$ at the point P .

(b) Using algebra, find the exact coordinates of P .

(4)



Question 2 continued



b. $2^x = 4^x - 6$

$$2^x = 2^{2x} - 6 \quad \leftarrow 4 = 2^2$$

$$2^{2x} - 2^x - 6 = 0$$

$$(2^x)^2 - 2^x - 6 = 0$$

let $y = 2^x$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3 \quad y = -2$$

$$2^x = 3 \quad \leftarrow \text{cannot log a negative number}$$

$$x = \log_2 3$$

$$P: (\log_2 3, 2)$$

Q2

(Total 8 marks)



3. (i) Prove that for all single digit prime numbers, p ,

$$p^3 + p \text{ is a multiple of } 10 \quad (2)$$

- (ii) Show, using algebra, that for $n \in \mathbb{N}$

$$(n+1)^3 - n^3 \text{ is not a multiple of } 3 \quad (3)$$

- i. We can use proof by exhaustion as there are only 4 single digit prime numbers

$p=2$	$p=3$	$p=5$	$p=7$
$2^3 + 2 = 10$	$3^3 + 3 = 30$	$5^3 + 5 = 130$	$7^3 + 7 = 350$ ✓ M1

Each case gives a multiple of 10. As 2, 3, 5, 7 are the only single digit primes, the statement has been proven true for all single digit primes ✓ A1

- ii. $(n+1)^3 - n^3$

$$= n^3 + 3n^2 + 3n + 1 - n^3 \quad \checkmark M1$$

$$= 3n^2 + 3n + 1 \quad \checkmark A1$$

$$= 3(n^2 + n) + 1 \Rightarrow \text{one more than a multiple of } 3, \text{ so not divisible by } 3 \text{ for any } n \in \mathbb{N} \quad \checkmark A1$$



Question 3 continued

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Q3

(Total 5 marks)



P 6 5 7 9 3 A 0 9 3 2

4. (a) Find, in ascending powers of x , up to and including the term in x^3 , the binomial expansion of

$$\left(2 + \frac{x}{8}\right)^{13}$$

fully simplifying each coefficient.

(4)

- (b) Use the answer to part (a) to find an approximation for 2.0125^{13}

Give your answer to 3 decimal places.

(3)

Without calculating 2.0125^{13}

- (c) state, with a reason, whether the answer to part (b) is an overestimate or an underestimate.

(1)

a. BINOMIAL EXPANSION FORMULA:

$$(a + bx)^n = a^n + \binom{n}{1} a^{n-1} (bx)^1 + \binom{n}{2} a^{n-2} (bx)^2 + \dots + \binom{n}{n-1} a^1 (bx)^{n-1} + (bx)^n$$

← This is the CHOOSE function on your calculator

$$\left(2 + \frac{x}{8}\right)^{13} = 2^{13} + \binom{13}{1} 2^{12} \left(\frac{x}{8}\right)^1 + \binom{13}{2} 2^{11} \left(\frac{x}{8}\right)^2 + \binom{13}{3} 2^{10} \left(\frac{x}{8}\right)^3 \quad \checkmark M1$$

$$8192 + 6656x + 2496x^2 + 572x^3$$

$$= 8192 + 6656x + 2496x^2 + 572x^3 \quad \checkmark A1 \quad \checkmark A1$$

✓ B1

b. Solve for x in:

$$2 + \frac{x}{8} = 2.0125$$

$$x = 0.1 \quad \checkmark B1$$

$$2.0125^{13} = 8192 + 6656(0.1) + 2496(0.1)^2 + 572(0.1)^3 \quad \checkmark M1$$

$$= 8683.132 \text{ (3dp)} \quad \checkmark A1$$

- c. Underestimate, because all terms are positive, so there would be more terms added to the truncated series

✓ B1



Question 4 continued

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Handwritten mathematical formulas and symbols are visible in the background, including:

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $E = mc^2$
- $a^2 + b^2 = c^2$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $(a+b)^2 = a^2 + 2ab + b^2$
- π
- $\sqrt{x}$
- α
- $\frac{1}{x}$
- $\frac{1}{y}$
- $\frac{1}{z}$
- $\frac{1}{w}$
- $\frac{1}{v}$
- $\frac{1}{u}$
- $\frac{1}{t}$
- $\frac{1}{s}$
- $\frac{1}{r}$
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- $\frac{1}{f}$
- $\frac{1}{e}$
- $\frac{1}{d}$
- $\frac{1}{c}$
- $\frac{1}{b}$
- $\frac{1}{a}$

(Total 8 marks)

Q4



5.

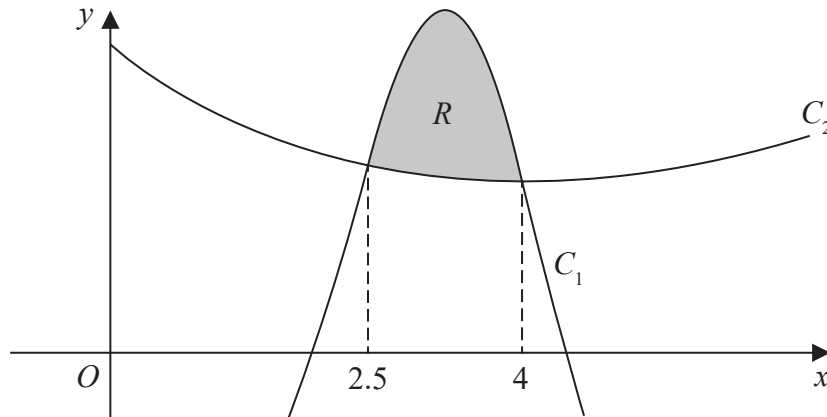


Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

- (a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1 , the line with equation $x = 2.5$, the x -axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \quad x > 0$$

- (b) Find $\int_{2.5}^4 (x^{\frac{3}{2}} - 3x + 9) dx$

(3)

The region R , shown shaded in Figure 2, is bounded by the curves C_1 and C_2

- (c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R .

(3)

a.

TRAPEZIUM RULE

$$\int_a^b y \, dx = \frac{1}{2} h [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})], \text{ where } h = \frac{b-a}{n}$$

$\frac{1}{2} h$ represents the width of each strip
 $(y_0 + y_n)$ height of the first and last strip respectively
 $2(y_1 + y_2 + \dots + y_{n-1})$ heights of strips in between the first and last
 n number of strips

Question 5 continued

$$h = \frac{4-2.5}{6}$$

From the table there are 6 strips

$$h = 0.25 \quad \checkmark \text{ 81}$$

$$A = \frac{1}{2} \times 0.25 \left[(5.453 + 5) + 2(7.764 + 9.375 + 9.964 + 9.367 + 7.626) \right] \quad \checkmark \text{ M1 } \checkmark \text{ A1}$$

$$A = 12.33$$

$$A = 12.33 \quad \checkmark \text{ A1}$$

$$b. \int x^{3/2} - 3x + 9 \, dx = \frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x + c$$

$$\frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x + c \quad \checkmark \text{ M1 } \checkmark \text{ A1 } \checkmark \text{ A1}$$

c. The area of R is just the area under C_1 - area under C_2 between $x=2.5$ and $x=4$

Area under C_1

$$C_1 = 12.33 \quad (\text{from part (a)})$$

Area under C_2

$$\left[\frac{2}{5}x^{5/2} - \frac{3}{2}x^2 + 9x \right]_{2.5}^4$$

$$= \frac{2}{5}(4)^{5/2} - \frac{3}{2}(4)^2 + 9(4) - \left(\frac{2}{5}(2.5)^{5/2} - \frac{3}{2}(2.5)^2 + 9(2.5) \right) \quad \checkmark \text{ M1}$$

$$= 24.8 - 17.077 \dots$$

$$= 7.722$$

$$\therefore \text{Area R} = 12.33 - 7.722 \quad \checkmark \text{ M1}$$

$$= 4.6 \quad (1 \text{ dp}) \quad \checkmark \text{ A1}$$

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Question 5 continued

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Q5

(Total 10 marks)



P 6 5 7 9 3 A 0 1 7 3 2

6. A circle has equation

$$x^2 - 6x + y^2 + 8y + k = 0$$

where k is a positive constant.

Given that the x -axis is a tangent to this circle,

- (a) find the value of k .

(3)

The circle meets the coordinate axes at the points R , S and T .

- (b) Find the exact area of the triangle RST .

(4)

- a. Subbing in $y=0$ (equation of line along x -axis):

$$x^2 - 6x + 0^2 + 8 \times 0 + k = 0$$

$$x^2 - 6x + k = 0 \quad \checkmark M1$$

Since x -axis is tangent to circle, there must only be 1 solution in $x^2 - 6x + k = 0$

DISCRIMINANT FORMULA

$$b^2 - 4ac = 0 \rightarrow \text{Repeated roots}$$

$$b^2 - 4ac > 0 \rightarrow \text{Two real roots}$$

$$b^2 - 4ac < 0 \rightarrow \text{No real roots}$$

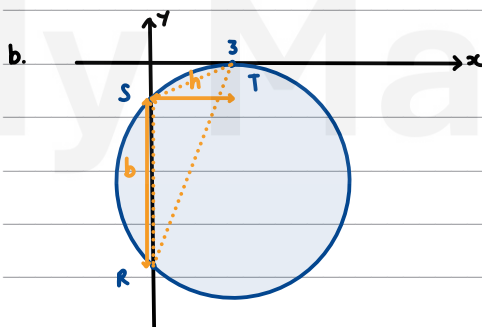
$$b^2 - 4ac = 0$$

$$(-6)^2 - 4(1)(k) = 0 \quad \checkmark M1$$

$$36 - 4k = 0$$

$$4k = 36$$

$$k = 9 \quad \checkmark A1$$



Point where circle meets x -axis (point T) = height

$$x^2 - 6x + 9 = 0 \quad \leftarrow k \text{ value from part (a)}$$

$$(x-3)^2 = 0$$

$$x = 3$$

$$\text{height} = 3 \quad \checkmark B1$$



Question 6 continued

Finding R and S.

Sub in $x=0$

$$0^2 - 6(0) + y^2 + y + 9 = 0$$

$$y^2 + y + 9 = 0$$

$$(y+4)^2 - 16 + 9 = 0$$

$$(y+4)^2 = 7$$

$$y+4 = \pm\sqrt{7}$$

$$y = -4 \pm \sqrt{7} \quad \checkmark M1$$

$$R: -4 - \sqrt{7} \quad S: -4 + \sqrt{7}$$

Distance between R and S = base

$$-4 - \sqrt{7} - (-4 + \sqrt{7}) = -2\sqrt{7}$$

$$\text{distance} = 2\sqrt{7}$$

$$\text{base} = 2\sqrt{7}$$

$$\text{Area of } \Delta RST = \frac{1}{2} \times 2\sqrt{7} \times 3 \quad \checkmark M1$$

$$= 3\sqrt{7} \quad \checkmark A1$$

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Question 6 continued

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Question 6 continued

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Q6

(Total 7 marks)



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7. (a) Given that

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

show that

$$2x^3 - 3x^2 - 30x + 56 = 0 \quad (4)$$

(b) Show that -4 is a root of this cubic equation. (2)

(c) Hence, using algebra and showing each step of your working, solve

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25) \quad (4)$$

a. $\log(a^b) = b \log(a)$ $\log(ab) = \log a + \log b$

$$3 \log_3(2x - 1) = 2 + \log_3(14x - 25)$$

$$3 \log_3(2x - 1) = \log_3(3^2) + \log_3(14x - 25) \quad \leftarrow 2 \text{ can be written as } \log_3(3^2)$$

$$\log_3[(2x - 1)^3] = \log_3[9(14x - 25)]$$

$$(2x - 1)^3 = 9(14x - 25)$$

$$8x^3 - 12x^2 + 6x - 1 = 126x - 225$$

$$8x^3 - 12x^2 - 120x + 224 = 0 \quad \div 4$$

$$2x^3 - 3x^2 - 30x + 56 = 0$$

b. Sub in $x = -4$ into equation in part (a)

$$2(-4)^3 - 3(-4)^2 - 30(-4) + 56 \quad \checkmark M1$$

$$= -128 - 48 + 120 + 56$$

$$= 0 \quad \checkmark A1$$

$\therefore$ Hence -4 is a root of the equation

c. Since $x = -4$ is a root, then $x + 4$ must be a factor. Do algebraic division to obtain the quadratic polynomial factor of $2x^3 - 3x^2 - 30x + 56$

$$\begin{array}{r} 2x^2 - 11x + 14 \\ x+4 \overline{) 2x^3 - 3x^2 - 30x + 56} \\ \underline{-(2x^3 + 8x^2)} \\ -11x^2 - 30x \\ \underline{-(-11x^2 - 44x)} \\ 14x + 56 \\ \underline{-(14x + 56)} \\ 0 \end{array}$$

$\Rightarrow 2x^3 - 3x^2 - 30x + 56 = (x+4)(2x^2 - 11x + 14) = 0 \quad \checkmark M1$

$0 = (x+4)(2x^2 - 11x + 14) \quad \checkmark A1$

$\downarrow$ factorise $\checkmark dM1$

$x+4=0 \quad 2x^2 - 11x + 14 = 0 \quad x-2=0$

$x=-4 \quad x = \frac{7}{2} \quad x=2$

Not defined for $x=-4$

as $3 \log_3(2(-4)-1) = 3 \log_3(-9)$

$\therefore x = \frac{7}{2}, x=2 \quad \checkmark A1$



Question 7 continued

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Question 7 continued

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Question 7 continued

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Q7

(Total 10 marks)



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8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (i) Solve, for $0 < \theta < 360^\circ$, the equation

$$3 \sin(\theta + 30^\circ) = 7 \cos(\theta + 30^\circ)$$

giving your answers to one decimal place.

(4)

- (ii) (a) Show that the equation

$$3 \sin^3 x = 5 \sin x - 7 \sin x \cos x$$

can be written in the form

$$\sin x (a \cos^2 x + b \cos x + c) = 0$$

where a , b and c are constants to be found.

- (b) Hence solve for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ the equation

$$3 \sin^3 x = 5 \sin x - 7 \sin x \cos x$$

(6)

i. $\textcircled{1} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$

$$3 \sin(\theta + 30^\circ) = 7 \cos(\theta + 30^\circ)$$

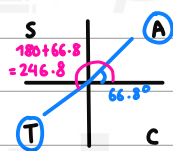
$$\textcircled{1} \quad \tan(\theta + 30^\circ) = \frac{7}{3} \quad \checkmark M1$$

$$\theta + 30^\circ = \tan^{-1}\left(\frac{7}{3}\right) \quad \checkmark M1$$

$$\theta + 30^\circ = 66.8^\circ, 246.8^\circ$$

$$\theta = 36.8^\circ, 216.8^\circ$$

$\checkmark A1$ $\checkmark A1$



← since $\tan(\theta + 30^\circ)$ is positive, we are interested in the

first and third quadrants (A and T respectively)

so solutions are 66.8 and $180 + 66.8 = 246.8$

ii. a. $\textcircled{2} \quad \sin^2 x = 1 - \cos^2 x$

$$3 \sin^3 x = 5 \sin x - 7 \sin x \cos x$$

$$3 \sin x \sin^2 x = 5 \sin x - 7 \sin x \cos x$$

$$\textcircled{2} \quad 3 \sin x (1 - \cos^2 x) = 5 \sin x - 7 \sin x \cos x \quad \checkmark M1$$

$$5 \sin x - 7 \sin x \cos x - 3 \sin x (1 - \cos^2 x) = 0$$

$$\sin x (5 - 7 \cos x - 3 + 3 \cos^2 x) = 0$$

$$\sin x (3 \cos^2 x - 7 \cos x + 2) = 0 \quad \checkmark A1$$



Question 8 continued

b. Using part (a):

$$\sin x (3\cos^2 x - 7\cos x + 2) = 0$$

$$\sin x = 0$$

$$x = 0 \quad \checkmark B1$$

factorise $\rightarrow 3\cos^2 x - 7\cos x + 2 = 0$

$$(3\cos x - 1)(\cos x - 2) = 0 \quad \checkmark M1$$

$$3\cos x - 1 = 0 \quad \cos x = 2 \quad \text{not possible as } \cos \text{ has range } -1 \text{ to } 1$$

$$\cos x = \frac{1}{3} \quad \checkmark M1$$

$$x = 1.23, -1.23 \quad \leftarrow \text{remember } \cos \text{ is symmetrical across the } y\text{-axis.}$$

$\checkmark M1$

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Question 8 continued

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Q8

(Total 10 marks)



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9.

- Bottom face
- Side faces

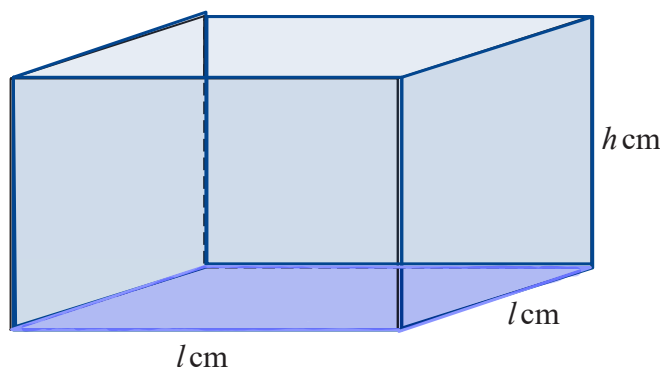


Figure 3

Figure 3 shows a sketch of a square based, open top box.

The height of the box is h cm, and the base edges each have length l cm.

Given that the volume of the box is $250\,000\text{ cm}^3$

- (a) show that the external surface area, $S\text{ cm}^2$, of the box is given by

$$S = \frac{250\,000}{h} + 2000\sqrt{h} \quad (3)$$

- (b) Use algebraic differentiation to show that S has a stationary point when $h = 250^k$ where k is a rational constant to be found. (5)

- (c) Justify by further differentiation that this value of h gives the minimum external surface area of the box. (2)

a. Remember that box has open top! So we don't add the area of top face

Since the equation we have to show does not include l , we have to get l in terms of h .

$$\text{Volume} = l \times l \times h$$

$$250\,000 = l^2 h$$

$$l = \frac{500}{\sqrt{h}} \quad \checkmark \text{ B1}$$

Surface area is just the sum of areas of faces of cuboid excluding the top face

Bottom face Side faces

$$S = l^2 + 4lh \quad \checkmark \text{ M1}$$

$$= \left(\frac{500}{\sqrt{h}}\right)^2 + 4\left(\frac{500}{\sqrt{h}}\right)h$$

$$= \frac{250\,000}{h} + \frac{2000h}{\sqrt{h}}$$



Question 9 continued

$$S = \frac{250000}{h} + 2000\sqrt{h} \quad \checkmark A1^*$$

b. Stationary point is where $\frac{dS}{dh} = 0$

$$S = \frac{250000}{h} + 2000\sqrt{h}$$

$$\frac{dS}{dh} = -\frac{250000}{h^2} + 1000h^{-1/2} \quad \checkmark M1 \quad \checkmark A1$$

$$-\frac{250000}{h^2} + 1000h^{-1/2} = 0 \quad \checkmark ddm \quad \times h^2$$

$$-250000 + 1000h^{3/2} = 0$$

$$1000h^{3/2} = 250000$$

$$h^{3/2} = 250 \quad \checkmark ddm1$$

$$h = 250^{2/3}$$

$$h = 250^{2/3} \quad \checkmark A1$$

c. Minimum is when $\frac{d^2S}{dh^2} > 0$

$$\frac{dS}{dh} = -\frac{250000}{h^2} + 1000h^{-1/2}$$

$$\frac{d^2S}{dh^2} = \frac{500000}{h^3} - 500h^{-3/2} \quad \checkmark M1$$

Subbing in $h = 250^{2/3}$

$$\frac{d^2S}{dh^2} = \frac{500000}{(250^{2/3})^3} - 500(250^{2/3})^{-3/2}$$

$$= 6$$

$\therefore$ Since $\frac{d^2h}{dh^2} = 6 > 0$, $h = 250^{2/3}$ gives the minimum value $\checkmark A1$

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Question 9 continued

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Q9

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

